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Method of Doppler Data Processing for Orbit Determination

Prepared by GILBERTO IALONGO
Space Physics Laboratory

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Laboratory Operations
THE AEROSPACE CORPORATION

Prepared for SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
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FOREWORD

This report is published by The Aerospace Corporation, El Segundo, California, under Air Force Contract No. F04701-69-C-0066. This work was done in support of orbit determination operations required by the Air Force Satellite Control, AFSCF.

This report, which documents research carried out from June 1969 to August 1969, was submitted for review and approval on 6 March 1970 to Lt. S. R. Weinstein (SMOND-1).

This report contains no classified information extracted from other classified documents.

Approved

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

S. R. Weinstein
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Orbit Determination Project Officer

ABSTRACT

A range rate expression is derived starting from a general relativistic Doppler shift formula. This expresssion differs from the one presently used by a term $F(\theta,\phi)$. The inclusion of this term, which vanishes for near-Earth geometries, allows range rate determination to better than 1 cm/sec.

CONTENTS

FIGURES

I. INTRODUCTION

A general relativistic formula for a two-day Doppler shift has been derived in a previous report.[1] Where the location of the ground based transmitter and receiver is the same, much of the general relativistic effect drops out and the formula can be written as

$$\frac{f_r}{f_t} = K \frac{1 - \frac{\vec{v}_s \cdot \hat{R}_t}{c}\left(1 + \frac{2m}{r_s}\right)}{1 - \frac{\vec{v}_t \cdot \hat{R}_t}{c}\left(1 + \frac{2m}{r_t}\right)} \cdot \frac{1 - \frac{\vec{v}_r \cdot \hat{R}_s}{c}\left(1 + \frac{2m}{r_t}\right)}{1 - \frac{\vec{v}_s \cdot \hat{R}_s}{c}\left(1 + \frac{2m}{r_s}\right)} \tag{1}$$

In Eq. 1, f_r and f_t are the received and transmitted frequencies, respectively, and K is the factor by which the transmitted frequency is multiplied at the satellite before being retransmitted back. The geocentric distances of the satellite and transmitter are r_s and r_t, respectively; $\vec{v}_t$, $\vec{v}_r$ and $\vec{v}_s$ are the velocity vectors of the transmitter, receiver and satellite respectively; and c is the velocity of light and $m \equiv GM_E/c^2$, where G is the gravitational constant and M_E the mass of the Earth. $\hat{R}_t$ and $\hat{R}_s$ are defined as

$$\hat{R}_t \equiv \frac{\vec{r}_s - \vec{r}_t}{\left|\vec{r}_s - \vec{r}_t\right|} \quad \text{and} \quad \hat{R}_s \equiv \frac{\vec{r}_r - \vec{r}_s}{\left|\vec{r}_r - \vec{r}_s\right|} \tag{2}$$

Even though the receiving and transmitting antennas are the same physical object, we still have $\vec{r}_t \neq \vec{r}_r$ and $\vec{v}_t \neq \vec{v}_r$. This is the result of using a stationary (inertial) coordinate system with the origin at the center of the Earth and measuring the coordinates of the antenna location at two different

times, i.e., the time of transmission and the time of reception of the signal. Since the Earth is moving, the coordinates of the antenna location in the inertial system are a function of time. In the limit of $\vec{r}_s \rightarrow \vec{r}_t$, for a near-Earth situation, we also have $\vec{r}_t \rightarrow \vec{r}_r$ and $\vec{v}_t \rightarrow \vec{v}_r$.

II. INTERPRETATION OF THE DOPPLER FORMULA

Eq. 1 is relativistically correct to orders of $(v/c)^3$ included and m/r is typically of the other of $(v/c)^2$. The present range rate determination method is depedent on the reading of a cycle counter. Truncation of one count produces an error of the order of 0.04 ft/sec $\approx$ 1 cm/sec. For this reason, in the expression for f_r/f_t, terms proportional to $(v/c)^3$, which bring corrections of the order of 10^{-5} cm/sec can be neglected; the terms in $(v/c)^2$ which bring corrections of the order of 1 cm/sec must be retained.

In accordance with the arguments presented in Reference 1, the distinction between $\vec{r}_t$ and $\vec{r}_r$ and $\vec{v}_t$ and $\vec{v}_r$ in the terms of order $(v/c)^2$ can be dropped without losing the desired accuracy. Eq. 1 therefore becomes

$$\frac{f_r}{f_t} = K\left[1 + \frac{1}{c}\left(\vec{v}_s \cdot \hat{R}_s + \vec{v}_t \cdot \hat{R}_t - \vec{v}_r \cdot \hat{R}_s - \vec{v}_s \cdot \hat{R}_t\right) + 2\,\frac{\dot{r}^2}{c^2}\right] \tag{3}$$

where $\dot{r} = \left(\vec{v}_s - \vec{v}_t\right) \cdot \hat{R}_t$ is the range rate.

$\hat{R}_s$ can be expanded[1] in terms of $\hat{R}_t$ to yield

$$\hat{R}_s = -\hat{R}_t + \frac{\omega\,\Delta t\; r_t}{R_t}\sin\theta\;\hat{R}_{\perp,t} \tag{4}$$

where $\hat{R}_{\perp,t}$ is the unit vector normal to $\hat{R}_t$, ω the angular frequency of the Earth and Δt the time elapsed between the transmission of a radio signal and its reception after retransmission from the satellite. θ is the angle between the velocity vector $\vec{v}_t$ and the displacement $\vec{R}_t$.

In order to develop a more suitable expression for the terms in c^{-1}, we note that the velocity of the transmitter is

$$\vec{v}_t = \vec{\omega} \times \vec{r}_t \tag{5}$$

and that the location of the receiver can be written in terms of $\vec{r}_t$ and $\vec{v}_t$ in the following form

$$\vec{r}_r = \vec{r}_t + \omega \, \Delta t \; r_t \hat{v}_t \tag{6}$$

Therefore

$$\vec{v}_r = \vec{\omega} \times \vec{r}_t + \omega \, \Delta t \; r_t \left(\vec{\omega} \times \hat{v}_t\right) \tag{7}$$

or

$$\vec{v}_r = \vec{v}_t + \omega^2 \, \Delta t \; r_t \hat{v}_{\perp,t} \tag{8}$$

where $\hat{v}_{\perp,t}$ is a unit vector normal to $\hat{v}_t$. Equations 4 and 8 can now be used to eliminate $\vec{v}_r$ and $\hat{R}_s$ in the terms in c^{-1}. The result is

$$\vec{v}_s \cdot \hat{R}_s + \vec{v}_t \cdot \hat{R}_t - \vec{v}_r \cdot \hat{R}_s - \vec{v}_s \cdot \hat{R}_t = -2\dot{r} + \frac{\omega \, \Delta t \; r_t}{R_t} \sin\theta \left(\vec{v}_s - \vec{v}_t\right) \cdot R_{\perp,t}$$
$$+ \omega^2 \, \Delta t \; r_t \hat{v}_{\perp,t} \cdot \hat{R}_t + 0\left[\ ^3\right] \tag{9}$$

The quantity of order ω^3 can be neglected in Eq. 9, and substitution in the expression for f_r/f_t gives,

$$\frac{f_r}{f_t} = K\left\{1 - 2\frac{\dot{r}}{c} + 2\left(\frac{\dot{r}}{c}\right)^2 + \frac{1}{c}\left[\frac{\omega\,\Delta t\, r_t \sin\theta}{R_t}\left(\vec{v}_s - \vec{v}_t\right)\cdot\hat{R}_{\perp,t} + \omega^2\,\Delta t\, r_t\hat{v}_{\perp,t}\cdot\hat{R}_t\right]\right\} \tag{10}$$

The relationship between the vector quantities appearing in Eq. 10 is shown in Fig. 1.

It is obvious in Fig. 1 that

$$\hat{v}_t = \cos\theta\,\hat{R}_t + \sin\theta\,\hat{R}_{\perp,t} \tag{11}$$

and

$$\vec{v}_t\cdot\hat{R}_{\perp,t} = v_t\cos\left(\frac{\pi}{2} - \theta\right) = v_t\sin\theta \tag{12}$$

$$\hat{v}_{\perp,t}\cdot\hat{R}_t = -\cos\left(\frac{\pi}{2} - \theta\right) = -\sin\theta \tag{13}$$

$$\vec{v}_s\cdot\hat{R}_{\perp,t} = v_s\cos\phi \tag{14}$$

Equation 10 can now be written in the form

$$\frac{f_r}{f_t} = K\left[1 - 2\frac{\dot{r}}{c} + 2\left(\frac{\dot{r}}{c}\right)^2 + \frac{\omega\,\Delta t\, r_t}{cR_t}\sin\theta\left(v_s\cos\phi - v_t\sin\theta\right) - \frac{\omega^2\Delta t\, r_t\sin\theta}{c}\right] \tag{15}$$

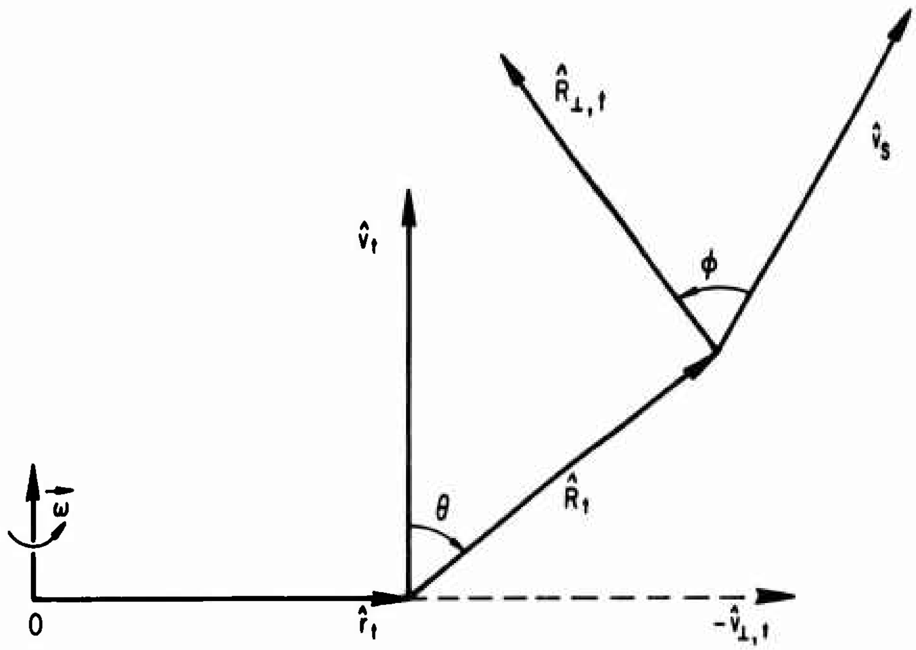

Fig. 1. Relationships Between the Vectors Involved in Eq. 10. $\vec{\omega}$ is Coming Out of the Page, the Other Vectors Lie on the Page

Using the fact that $\omega^2 r_t = \omega(\omega r_t) = \omega v_t$ and rearranging, we have

$$\frac{f_r}{f_t} = K\left[1 - 2\,\frac{\dot{r}}{c} + 2\left(\frac{\dot{r}}{c}\right)^2 - \frac{\omega\,\Delta t\,v_t}{c}\left(1 + \frac{r_t}{R_t}\sin\theta - \frac{r_t\,v_s}{R_t\,v_t}\cos\phi\right)\sin\theta\right] \tag{16}$$

or

$$\frac{f_r}{f_t} \equiv K\left[1 - 2\,\frac{\dot{r}}{c} + 2\left(\frac{\dot{r}}{c}\right)^2 - F(\theta,\phi)\right] \tag{17}$$

which defines $F(\theta,\phi)$.

Equation 17 differs from the formula which is being presently used[2] in the SGLS range-rate processing by the added term $F(\theta,\phi)$. This term is an effect of the finite time needed for a radio signal to complete the trip to the satellite and back. For near-Earth orbits, $\Delta t \to o$ and $F(\theta,\phi) \to o$ as well. Because of the linear dependence on $\sin\theta$, it can be seen that $F(\theta,\phi) = o$ when $\theta = o$. This is the case when the satellite is on the horizon. The reason for this is that in this limit $\hat{R}_s \to -\hat{R}_t$. $F(\theta,\phi)$ must also vanish when the satellite is on a circular synchronous orbit. This is easily checked by verifying that under such conditions, the quantity $f(\theta,\phi)$ is zero. $f(\theta,\phi)$ is defined as

$$f(\theta,\phi) = 1 + \frac{r_t}{R_t}\sin\theta - \frac{r_t\,v_s}{R_t\,v_t}\cos\phi \tag{18}$$

The relations between the quantities involved in the case of a circular orbit for the satellite is illustrated in Fig. 2 where it is shown that

$$r_s = r_t\cos\left(\frac{\pi}{2} - \theta - \phi\right) + R_t\cos\phi \tag{19}$$

or

$$r_s = r_t \sin (\theta + \phi) + R_t \cos \phi \tag{20}$$

The constraint that the satellite be on a geosynchronous orbit translates into the following equality

$$\omega = \frac{v_t}{r_t} = \frac{v_s}{r_s} \tag{21}$$

where ω is now the angular velocity of both a point on the Earth and of the satellite. To show that under the constraints of Fqs. 20 and 21, $f(\theta,\phi)$ reduces to zero, Eq. 18 can be rewritten in the form

$$f(\theta,\phi) = (R_t v_t + r_t v_t \sin \theta - r_t v_s \cos \phi)/R_t v_t \tag{22}$$

Use of Eqs. 20 and 21, yields

$$f_c(\theta,\phi) = \left[R_t v_t + r_t v_t \sin \theta - r_t v_t \sin (\theta + \phi) \cos \phi - R_t v_t \cos^2 \phi\right] / R_t v_t \tag{23}$$

where the subscript c indicates that the constraints have been used. Carrying out the algebra gives

$$f_c(\theta,\phi) = v_t \sin \phi \left[R_t \sin \phi - r_t \cos (\theta + \phi)\right] / R_v v_t \equiv 0 \tag{24}$$

where the last equality follows immediately with reference to Fig. 2.

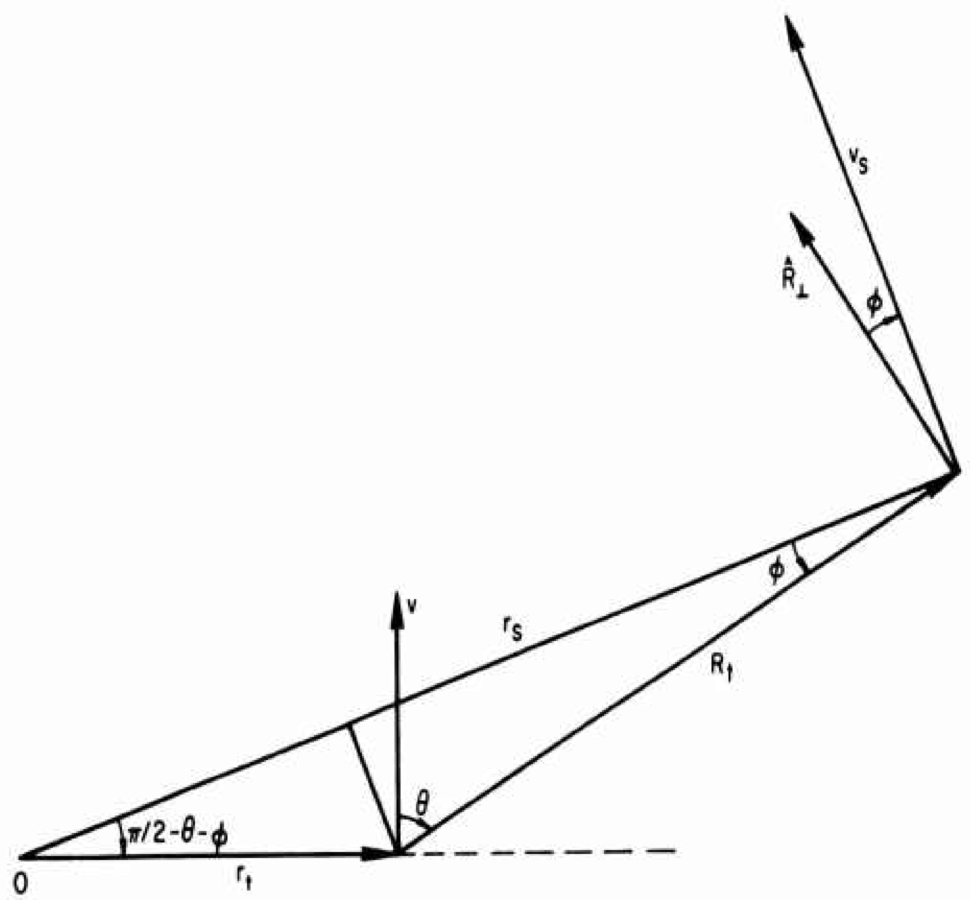

Fig. 2. Relationships Between the Vectors and Angles in the Case of a Circular Satellite Orbit

III. APPLICATION TO RANGE RATE DETERMINATION

Equation 17, the expression for the Doppler shift, can be used to compute the range rate of a satellite. In the present system, an input frequency, f_i, is used where

$$f_i \equiv \frac{5}{4} f_t - f_r \tag{25}$$

is fed to a counter N_1. In terms of Eq. 17, the input frequency becomes

$$f_i = \frac{f_t}{205}\left[\frac{1}{4} + 512\,\frac{\dot{r}}{c}\left(1 - \frac{\dot{r}}{c}\right) + 256\ F(\theta,\phi)\right] \tag{26}$$

where K has been set equal to 256/205.

The count interval is preset to last until the N_1 counter has accumulated $N_1 = 1{,}048{,}574$ cycles. A second counter, N_2, is slaved to start and stop with the N_1 counter and its input is defined by

$$N_2 = \frac{f_t}{64}\left[\frac{N_1}{\frac{1}{\delta t}\int_{t_o}^{t_o+\delta t} f_i\, dt}\right] - \frac{205}{16} N_1 \tag{27}$$

where the integral is carried over the count interval δt. δt is of the order of 0.5 sec. It should be noted that, in general, because of the presence of the term $F(\theta,\phi)$ in Eq. 26, the N_2 count is no longer zero when $\dot{r} = 0$.

Substitution of Eq. 26 into Eq. 27, and rearrangement of the terms to have N_1 and N_2 appear on one side and the satellite parameters on the other side, yields

$$\frac{1}{\delta t}\int_{t_o}^{t_o+\delta t}\left[\left(\dot{r} - \frac{\dot{r}^2}{c}\right) + F(\theta,\phi)\,\frac{c}{2}\right]dt = -\frac{cN_2}{2048N_2 + 26240N_1} \cdot \tag{28}$$

Since N_1 is a fixed number and N_2 is a measured quantity, the right hand side of Eq. 28 is determined. If we replace $\dot{r}$ with $\langle\dot{r}\rangle$ i.e., if we substitute the average value of the range rate over the count interval in place of the instantaneous range rate, and use the calculated satellite ephemeris data for the quantities appearing in $F(\theta,\phi)$, Eq. 28 will provide a way to compute $\langle\dot{r}\rangle$. It should be noted that since the term $F(\theta,\phi)\,\frac{c}{2}$ is of the order of $\dot{r}^2/c \ll \dot{r}$, the knowledge of the quantities appearing in $F(\theta,\phi)$ need not be extremely accurate. In partiuclar, $F(\theta,\phi)$ can be evaluated at any time during the count interval and can be treated as a constant. While the replacement $\dot{r} \rightarrow \langle\dot{r}\rangle$ is strictly correct for the term $\int_{t_o}^{t_o+\delta t}\dot{r}\,dt$, it is not rigorous for the term $\int_{t_o}^{t_o+\delta t}\dot{r}^2/c\,dt$ and in effect it amounts to the replacement

$$\int_{t_o}^{t_o+\delta t}\frac{\dot{r}^2}{c}\,dt \rightarrow \int_{t_o}^{t_o+\delta t}\langle\dot{r}\rangle\,\frac{\dot{r}}{c}\,dt \tag{29}$$

This is permissible because the quantity on the right side of Eq. 28 is very nearly the range rate.

Using the definition

$$\mathrm{RRN} \equiv - \frac{cN_2}{2048N_2 + 26240N_1} \tag{30}$$

and the replacements suggested above, the following quadratic equation for $<\dot{r}>$ is obtained

$$<\dot{r}>^2 - c <\dot{r}> + c\left[\mathrm{RRN} - F(\theta,\phi)\frac{c}{2}\right] = 0\,. \tag{31}$$

The physically acceptable solution of Eq. 31 is

$$<\dot{r}> = \frac{\mathrm{RRN} - F(\theta,\phi)\frac{c}{2}}{1 - \left[\mathrm{RRN} - F(\theta,\phi)\frac{c}{2}\right]/c} \tag{32}$$

where terms of the order of $\left[\mathrm{RRN} - F(\theta,\phi)\frac{c}{2}\right]^3/c^2 < 10^{-5}$ cm/sec have been neglected.

All the quantities on the right hand side of Eq. 32 are either measured or calculated and an accurate determination of the average range rate over a count interval is possible. Non-relativistic calculations are adequate.

IV. CONCLUSIONS

The two-way Doppler shift has been seen to be independent of general relativity effects to a high degree of approximation. On the other hand, corrections of the order of magnitude of 1 cm/sec have to be made for satellites at geosynchronous altitude. The nature of the correction is geometrical, and is needed because the orientation of the radius vector joining the transmitting station and the satellite is a function of time and a finite length of time elapses between the instant in which a radio signal is transmitted and received back on Earth. The range rate obtainable through Eq. 32 is averaged over a period of about 0.5 sec and is accurate to order of 10^{-5} cm/sec.

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13 ABSTRACT

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